

Texture Feature Classification of Liver Sonography Using Fuzzy Similarity Measures

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Abstract: It is difficult to differentiate diffuse liver diseases, namely cirrhotic and fatty livers from normal one by visual inspection from the ultrasound images. The need for computerized tissue characterization is thus justified to assist quantitatively the sonographer for accurate differentiation. In this paper a novel approach of tissue characterization using pattern recognition techniques is developed. Textural analysis methods based on co-occurrence matrix and gray-level gradient variations were applied to extract quantitative parameters for over 150 cases of three liver pathologies namely cirrhotic, fatty and normal livers. In addition to these textural feature descriptors an attenuation and speckle parameters were computed from the B-mode images. A fuzzy similarity measures as an approximate reasoning technique of matching between an unknown case defined by a feature vector and a family of prototypes were used for the classification steps. Finally we tested different textural methods and we could obtain a good results ranging from 80-95% of sensitivities and specificity for different liver pathologies.

Keywords: Ultrasound Texture Classification of Liver, Co-occurrence matrix parameters, gradient type variations parameters, attenuation, and speckle parameters, computerized medical diagnosis.

1. INTRODUCTION

Pulsed-echo ultrasound is a non-invasive technique capable of visualizing an internal structure of soft tissues and as such it is considered to be an extremely important and valuable tool of medical diagnosis. The physician has rely on detection of inhomogeneities between echo amplitudes received from the neighboring areas of the image. Such an approach is, of course, subjective and consequently problematic in itself. Moreover, in certain cases the disease attacks the entire tissue area, say, entire liver (diffuse liver diseases). Then, the ultrasonic image will be homogeneous (see figure 1), and as a result the diagnosis is sometimes difficult [1-8].

Visual criteria for diagnosing diffused liver diseases are in general confusing and highly subjective because they depend on the sonographer to observe certain textural characteristics from the image and compare them to those developed for different pathologies to determine the type of the disease. Moreover, some of the diseases are highly similar in their diagnostic criteria, which tend to confuse the sonographers even more.

The quantitative analysis of using ultrasound signals as an aid to the diagnosis of diffuse disease has been described by many researchers[2-12].

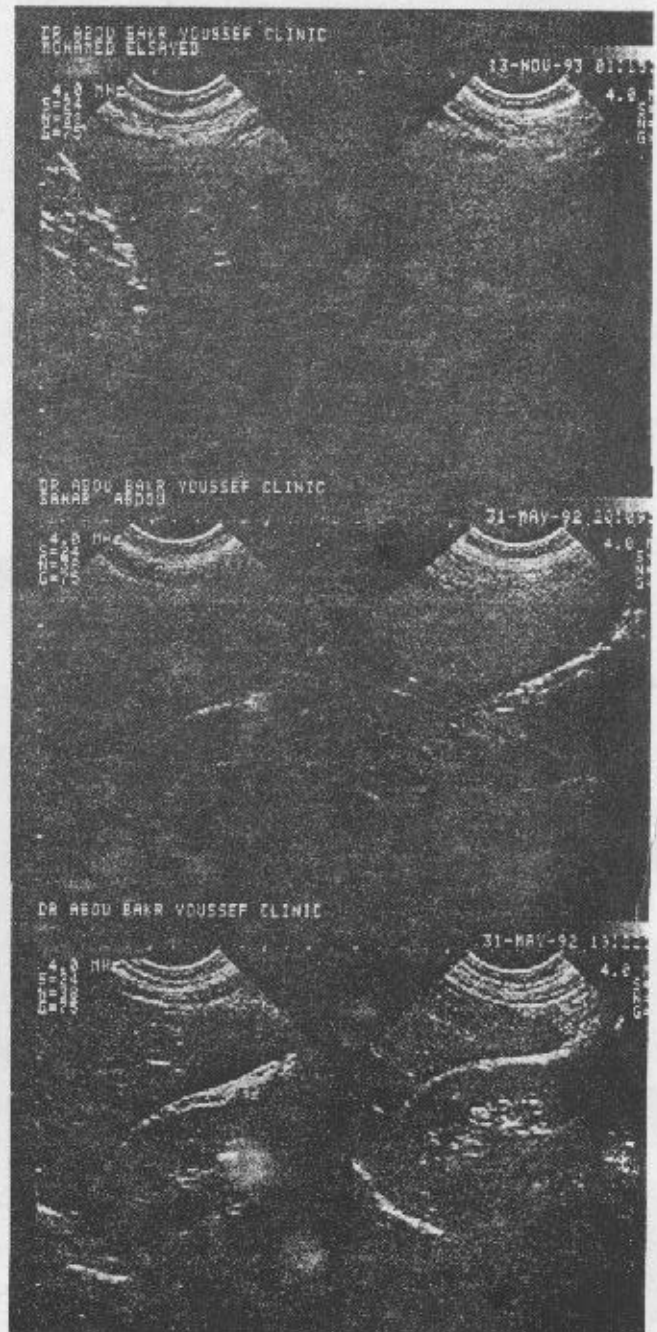


Figure 1 : Normal, Fatty, and Cirrhotic B-mode ultrasound images acquired at 4 MHz.

The field of fuzzy logic and fuzzy similarity [13-24] measure and its application in object matching is an active area of research. Many techniques of pattern matching are developed using fuzzy logic and the approximate reasoning. Mixed systems of neural network and fuzzy logic have also developed [20] for a wide range of applications. Very often, one is faced with a nontrivial case of partial matching, matching that returns with a degree of matching lying in a unit interval. In matching procedures, a concept of similarity is a significant one. As said previously, the matching procedure implies a degree of matching reached for two fuzzy quantities.

Unfortunately, nothing is known about certainty (or uncertainty) of the result obtained. We start with recalling some measures that are used for matching purposes. The main role of this section is to underline some of their characteristic features and highlight existing shortcomings.

2. MEASURES OF EQUALITY BETWEEN TWO FUZZY QUANTITIES

In this section, we will summarize some existing approaches that are useful for determination of a degree of equality (degree of matching) for two fuzzy quantities. Let us focus our attention on the comparison of two fuzzy sets A and B defined in the same universe of discourse X , say $A, B: X \rightarrow [0,1]$.

2.1 Distance Measure

A broad class of measures of equality is based on distance measure. Usually, a general form of Minkowski r -metric is given as:

$$d_n(A, B) = \left(\int_{-x}^{\infty} |A(x) - B(x)|^r dx \right)^{1/r} \quad r \geq 1 \quad (1)$$

2.2 Set-Theoretic considerations

The second class of measures of equality originates from some basic set-theoretic considerations.

- Based upon the dissimilarity measure defined as the ratio: $\text{Card}(A \cap B) / \text{Card}(A \cup B)$ (2)
- Possibility measure of two fuzzy sets. The measure describes the highest degree to which these two fuzzy quantities A and B overlap.

$$\mathcal{T}(A, B) = \sup_{x \in X} [\min(A(x), B(x))] \quad (3)$$

2.3 Logical framework.

The third way of dealing with the comparison of two fuzzy quantities is performed in a logical framework. One among well-known approaches in this group refers to linguistic evaluation of two fuzzy quantities that leads directly to notions of fuzzy logic (a so-called fuzzy truth values). For a certain element of the universe of discourse X a degree of equality [23,24] of a and b , $a, b, \in [0,1]$ is equal to

$$a \equiv b = \{(a \rightarrow b) \wedge (b \rightarrow a) + (\bar{a} \rightarrow \bar{b}) \wedge (\bar{b} \rightarrow \bar{a})\} \quad (4)$$

Here \wedge stands for minimum, \rightarrow forms an implication, where $\bar{a} = 1-a$. Then applying conjunctions known in fuzzy sets, the aforementioned formula is translated into the form plausible for computational purposes. Simply speaking the implication \rightarrow is modeled by various pseudo complements induced by corresponding t -norms e.g., for the t -norm [22] specialized as minimum reads as:

$$a \equiv b = \begin{cases} (1 + b - a) & \text{if } a > b \\ 1 & \text{if } a = b. \\ (1 + a - b) & \text{if } a < b \end{cases} \quad (5)$$

For another t -norm specialized as product we get

$$a \rightarrow b = \min(1, b/a)$$

and finally

$$a \equiv b = [(a \rightarrow b)(b \rightarrow a) + ((1-a) \rightarrow (1-b))(1-b \rightarrow (1-a))] = \begin{cases} [b/a + (1-a)/(1-b)], & \text{if } a > b \\ 1, & \text{if } a = b. \\ [a/b + (1-b)/(1-a)], & \text{if } a < b \end{cases} \quad (6)$$

The last method of matching of two fuzzy quantities is closely related to an essence of computations with fuzzy sets. Therefore, in further discussion we will concentrate ourselves on studies on the equality index as given by method 3. Additionally this third approach enables us to perform a point wise matching process. In the case of the third type of these measures it is sometimes of interest to have a mechanism within which one combines the grades of equality to get a single number specifying an overall characterization of equality of the fuzzy set. At least four basic methods are often utilized and we will add to this list the fuzzy integrals method and we will discuss it later.

A maximal value among the degree of equality is taken

A maximal value of the degrees of equality is considered.

Averaging way of aggregation; degrees of equality are averaged.

Fuzzy integrals method [22-24]

Each of the previously listed methods of aggregation leads to a point characterization. A significant amount of information is lost. Therefore, it is of interest to aggregate them accordingly to particular application needs.

2.4 Fuzzy measure

When we consider a certain set X , the function g that makes subset E and F correspond to the values in the interval $[0,1]$ are called fuzzy measures [24] if they have the following properties:

$$(1) \quad g(\emptyset) = 0, \quad g(X) = 1 \quad (7)$$

$$(2) \quad \text{If } E \subset F, \quad g(E) \leq g(F) \quad (8)$$

(3) If $E_1 \subseteq E_2 \subseteq \dots$ or $E_1 \supseteq E_2 \supseteq \dots$

$$\lim_{n \rightarrow \infty} g(E_n) = g\left(\lim_{n \rightarrow \infty} E_n\right) \quad (9)$$

2.5 Fuzzy integrals

The fuzzy integral [22-24] of function $h: X \rightarrow [0,1]$ on $E \subseteq X$ by fuzzy measure g is defined as follows:

$$h(x) \circ g = \frac{\max_{E \subseteq V} \left[\min_{x \in E} (h(x)) \wedge g(E) \right]}{1} \quad (10)$$

3. PROPOSED METHOD FOR PATTERN MATCHING

Many technique for pattern matching and classification using fuzzy logic has been proposed, and now used in many application such as speech and character recognition's, medical diagnosis [15-17] and decision making. In the following paragraph we will introduce a proposed method for medical diagnosis based on the similarity measure of the unknown case and the sets of a prototypes from a known cases.

Give a vector $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

where n in the dimension of the vector X and the number of the classification parameters (features) in the system see figure 2.

x_i denotes the measured i th feature of the event, and X represented as a point in n dimension vector space Ω_X consisting of m ill defined pattern classes $C_1, C_2, \dots, C_j, \dots, C_m$ let $R_1, R_2, \dots, R_j, \dots, R_m$ be the reference vectors where R_j associated with C_j containing h_j number of prototypes such that

$$R_j^{(i)} \in R_j, \quad i=1,2,\dots,h_j \quad (11)$$

The pattern X can then be assigned to be member of that class if it shows maximum similarity to this class.

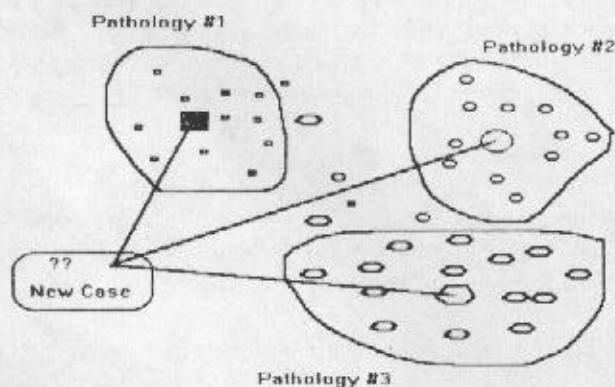


Figure 2: Illustration of the Idea of Diagnosis in the Parameter Space.

3.1 Fuzzification process.

Assume each feature as a linguistic variable has a number of fuzzy values e.g. High, Med, Low, and all the linguistic variables has the same number of fuzzy values. The fuzzification is done by getting the value of the membership functions, so obtaining a fuzzification matrix.

$$N = \text{Fuzz}(X) = \begin{bmatrix} x_{11} & \dots & x_{1j} & \dots & x_{1z} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ & & x_{ij} & & \\ & & \vdots & & \\ x_{z1} & & & & x_{zj} \end{bmatrix} \quad (12)$$

where z is the number of the linguistic values for the linguistic variables and

$$x_{ij} = F_{ij}(x_i) \quad (13)$$

where F_{ij} is the membership function of the linguistic value i for the fuzzy value j .

We do the fuzzification for the X and all R_j .

3.2 Similarity Measures

So the problem now is how to measure the similarity between N and R_j^i and obtain the over all similarity of this X and the other classes C_j represented by the R_j prototypes. As described in the previous sections that many technique can be used as, distance measure, from fuzzy set theories, linguistic evaluations. We will use the linguistic evaluation of two fuzzy quantities.

$$a = b =$$

$$\frac{1}{2} \{ (a \rightarrow b) \wedge (b \rightarrow a) + (\bar{a} \rightarrow \bar{b}) \wedge (\bar{b} \rightarrow \bar{a}) \} \quad (14)$$

if the implication chosen \min so the above equation can be read as:

$$a = b = \begin{cases} [b/a + (1-a)/(1-b)], & \text{if } a > b \\ 1, & \text{if } a = b \\ [a/b + (1-b)/(1-a)], & \text{if } a < b \end{cases} \quad (15)$$

so given N , $\text{Fuzz}(R_j^i) = R_j^i$ by using the linguistic evaluation obtain the similarity matrix S_j^i .

$$N = \begin{bmatrix} X_{11} & \dots & X_{1j} & \dots & X_{1n} \\ \vdots & & \vdots & & \vdots \\ & & X_{ij} & & \\ \vdots & & \vdots & & \vdots \\ X_{z1} & & & & X_{zn} \end{bmatrix}$$

$$R_j^i = \begin{bmatrix} r_{11} & \dots & r_{1j} & \dots & r_{1n} \\ \vdots & & \vdots & & \vdots \\ & & r_{ij} & & \\ \vdots & & \vdots & & \vdots \\ r_{z1} & & & & r_{zn} \end{bmatrix}$$

$$S_j^i = \begin{bmatrix} s_{11} & \dots & s_{1j} & \dots & s_{1n} \\ \vdots & & \vdots & & \vdots \\ & & s_{ij} & & \\ \vdots & & \vdots & & \vdots \\ s_{z1} & & & & s_{zn} \end{bmatrix}$$

where

$$s_{ij} = x_{ij} \equiv r_{ij}$$

Two methods can be used to get the similarity index

between the N and R_j^i

The first is to obtain the similarity vector H as follow:

$$H_j^i = \left[\frac{1}{z} \sum_{i=1}^z f(s_{z1}), \dots, \frac{1}{z} \sum_{i=1}^z f(s_{zn}) \right] = [h_1, \dots, h_n] \quad (20)$$

where f is a suggested function for re-weighting the linguistic evaluation, its' effect to increase the weight of the similar values and decrease the weight for dissimilar values. The f function could be any function to increase the degree of the similarity if it exceeds a certain threshold, or to decrease the weight of this number if the linguistic evaluation is less a certain threshold.

Many function can be used to do this mission, such as:

a) Sigmoid function

$$f(\alpha) = \frac{1}{1 + e^{-(\alpha - \theta)}} \quad (21)$$

where θ is the selected threshold.

b) Hard threshold

$$f(\alpha) = \begin{cases} 1 & \text{if } \alpha \geq \theta \\ 0 & \text{if } \alpha < \theta \end{cases} \quad (22)$$

c) S function

$$S(u) = \begin{cases} 0 & u \leq \alpha \\ 2 \left[\frac{(u - \alpha)}{(\gamma - \alpha)} \right]^2 & \alpha \leq u \leq \beta \\ 1 - 2 \left[\frac{(u - \gamma)}{(\gamma - \alpha)} \right]^2 & \beta \leq u \leq \gamma \\ 1 & u \geq \gamma \end{cases} \quad (23)$$

obtain the similarity index S_j^i

$$S_{H_j^i}^i = \frac{1}{n} \sum_n h_n \quad (24)$$

which represents how similar is this unknown case to the category j , prototype i .

The second method by using the fuzzy integral. Where h represents the similarity function and g represents a simple fuzzy measure which is the cardinality of the set E , $E \in X$ and X is the power set of the X .

$$hg = h(x) \circ g = \max_{E \in X} \left[\min_{x \in E} (h(x)) \wedge g(E) \right] \quad (25)$$

If we apply the fuzzy measure described before to the S_j^i rows.

$$S_j^i = \begin{bmatrix} R_1 \\ \vdots \\ R_z \end{bmatrix} \quad \text{where } R_i = [s_{i1}, \dots, s_{in}] \quad (26)$$

$$HG_j^i = \begin{bmatrix} hg_1 \\ \vdots \\ hg_z \end{bmatrix} \quad (27)$$

where hg_i is the fuzzy integral of the row i in the S_j^i matrix.

3.3 Aggregation methods

Many criteria can be selected to get the similarity between X and the category j , such as follows:

$$a) S_j = \max_i S_j^i \quad (28)$$

$$b) S_j = \min_i S_j^i \quad (29)$$

$$c) S_j = \frac{1}{I} \sum_i S_j^i \quad (30)$$

note that S_j^i can be S_G^i or S_H^i

4. ULTRASOUND IMAGE ACQUISITION AND FEATURES EXTRACTION

In the data acquisition system, the video output of a Kretz-320 mechanical sector ultrasound scanner was connected to a Matrox PIP-512 frame grabber card on an IBM-586 PC. The image is captured in 512X512 pixels, the

resolution is 8 bits/pixel. A s/w was developed to define the ROI and to extract all the aforementioned parameters (image analysis) [7-9].

To obtain a reproducible results, the following parameters were standardized for all tissue characterization parameters [7-9]:

1-Ultrasound machine settings

E.g., TGC, FOCUS, FREQUENCY, and ZOOM controls, which can change the overall image gain and produce zooming effects and hence deviates the image statistics in an unpredictable way. Moreover, the frequency of ultrasound waves used must be the same since the attenuation is frequency dependent.

2-ROI shape and size

To obtain a reliable statistics, the number of pixels in the ROI must be at least 1000 pixels (32 pixel per centimeter). The shape of the box is taken to be square.

4.1 Quantitative features

The quantitative parameters measured for ultrasound tissue characterization are broad categories extracted from pulse-echo data (gray scale B-mode image). There are more than 40 different parameters that extracted from the pulse echo data and are correlated with the pathology of the case. The categories of parameters are [5,6,8,10,11]:

1-Histogram parameters:

These are mean gray level (MGL), gray level variance (VAR), signal to noise ratio (MGL/VAR), skewness, curtosis and five of the relevant gray level histogram percentiles.

2-Co-occurrence matrix parameters:

These are contrast (CON), entropy (ENT), correlation (COR), and angular second moment (ASM).

3- First order gradient Parameters:

These are Absv_{aver}, Absv_{dir}, Absv_{var}, Absv_{var}, Most Dominant Edge Direction (MDE), Relative frequency of the most dominant edge (RFMDE)

4-Greylevel runlength matrix parameters:

These are Run percentage (RPER), Long-Run emphasis (LREM), Greylevel distribution (GDIST), Runlength distribution (RLDIST).

5-Texture feature descriptors (TFD) parameters:

These Coarseness (coarse), Homogeneity (HOM), Mean Convergence (MC), Variance (Var), Entropy (Entropy), Runlength Density (RLD), Regularity (Regularity), Grey level Resolution Similarity (GLRS).

6-Speckle Parameters:

These are mean scatterer separation (d), diffuse and specular scatterer intensity (I_d, I_s), specular standard deviation (σ_s) and a few other related parameters.

7-Acoustical parameters:

These are attenuation coefficient (ATTEN α) and the backscattering coefficient (BSC μ).

The sum of all these parameters may exceed 40. These parameters were evaluated using correlation measurements in order to have a reduced set of uncorrelated parameters, and to mark those parameters which correlate the strongest to the different pathologies [7,8].

The clustering of the three pathologies was previously done using statistical methods (k nearest neighbor), maximum likelihood method [6] neural networks [7,9] and Fuzzy logic by using fuzzy rules [12].

4.2 Features membership functions type and selection

Assume the domain intervals for each parameter, where the domain interval of a variable means that most probably this variable will lie in this interval (the value of the variable is allowed to be outside this domain). Divide each domain interval into three regions denoted by High, Low, and Med. Assign each region a certain fuzzy membership function. We have chosen three forms of membership functions the first is the triangle form, the second is the trapezoidal form and the third is bell form. The equation of the bell form used in the analysis is as follow:

$$\mu_S(s) = e^{-1/(s-\bar{s})^2/2\sigma^2} \quad (31)$$

where μ_S denotes the membership function of a fuzzy value.

Choosing the fuzzy singleton (\bar{S}) for each fuzzy set depends on two criteria: 1- statistical basis 2- expert knowledge. The bell form of the membership function given above is taken for the fuzzy value Med. For the Low value if $s < \bar{S}$ then μ_S equals to 1. For the High value if $s > \bar{S}$ then μ_S equals to 1.

Since we have only three pathologies and the size of the input space is 8 or 15, we have chosen only three regions for each variable because the high resolution is not required in this case to take a decision. The epsilon-completeness [12] is chosen to be equal to the crossover point as shown in figure 3 of membership functions for the Entropy.

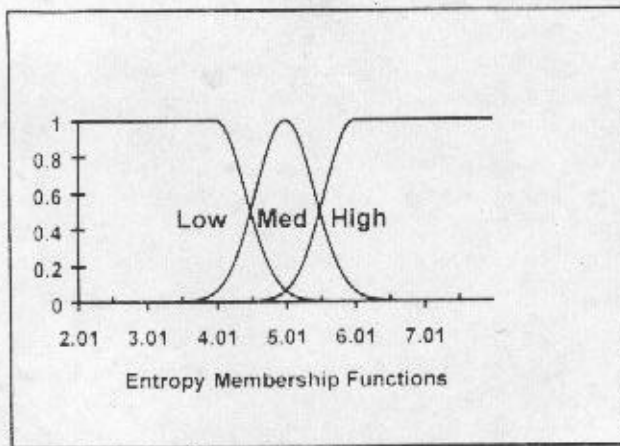


Figure 3: Example for membership grades of the Entropy

5. RESULTS

The image is quantitatively analyzed for the significant parameters of the reduced set. The significant parameters were classified into two types. Type one included the mean grey level (MGL), the grey level variance (VAR) extracted from the histogram parameters, the contrast (CON), Entropy (ENT), Angular second moment (ASM), and Correlation (CORR) extracted from the co-occurrence matrix parameters, MDE, RFMDE extracted from the first order gradient parameters, RPER, LREM, GDIST, RLDIST extracted from the runlength matrix, speckle separation, attenuation coefficient, and backscatt. coefficient.

Type two included these Coarseness(coarse), Homogeneity(HOM), Mean Convergence(MC), Variance(Var), Entropy(Entropy), Runlength Density(RLD), Regularity(Regularity), Greylevel Resolution Similarity(GLRS) extracted from the texture feature descriptors parameters. A needle BIOPSY is obtained for every patient. The decision was made based on the history information, laboratory, pathological (BIOPSY), clinical measurements, and clinician experience.

The aforementioned protocol was done for a set consisting of greater than 150 cases for the three pathologies: Normal, Fatty, and Cirrhotic livers.

The total number of cases used is greater than 150 cases (we acquire images for patients if it is fully clinically and pathologically investigated) for the three classes. Each class contains approximately 50 cases.

Using the fuzzy similarity techniques described above to get the degree of similarity between an unknown case represented by the vector X in the 8th or 15th dimensional space and the sets of prototypes. We have tested the system using cases greater than 50 unknown cases and the technique showed a very good results that match up with the clinical and pathological investigations (BIOPSY) see tables 1,2.

The definition of *sensitivity* and *specificity* is given as follows:

A	B
C	D
A+C	B+D

$$\text{Sensitivity} = \frac{A}{A+C} \times 100$$

$$\text{Specificity} = \frac{D}{B+D} \times 100$$

Where A : is the true positive, B : is the false positive, C : is the false negative, D : is the true negative, A+C : is the total positive, B+D : is the total negative.

If we found a high similarity between the test cases that are fully investigated and the family of prototypes, we append this case to the prototypes family. If the case is fully investigated, it is appended directly to our prototypes.

Pathology	Training Set	Test Set
Specificity	100%	92%
Sensitivity for Cirrhosis	100%	95%
Sensitivity for Fatty	100%	92.5%

Table 1: Error Using Similarity Measures (fatty, cirrhosis and normal liver pathologies) for the 15 features of type 1.

Pathology	Training Set	Test Set
Specificity	100%	89%
Sensitivity for Cirrhosis	100%	92%
Sensitivity for Fatty	100%	91%

Table 2: Error Using Similarity Measures (fatty, cirrhosis and normal liver pathologies) for the 8 features of type 2.

6. DISCUSSION AND CONCLUSION

The results of this work revealed the potential value for considering the idea of fuzzy similarity measures in tissue characterization of diffused liver diseases. We can apply this for most of the soft tissues that their diffuse diseases are confusing like liver, spleen and kidney diseases.

This potential value could be used for an on-line diagnosis of the pathology, and minimize the risk of taking needle Biopsy from the patient. The results of this work was compared to the other techniques used for tissue classification as statistical similarity [7], neural network [7-9], fuzzy logic rules methods [12] and the results showed an excellent results for correct diagnosis. The technique is very superior to that of statistical classification and

maximum likelihood classification done in [6], it has a much better accuracy since here we don't assume any a priori parametric distribution for the parameters as in reference [6].

For the sonographers in Egypt it would be difficult to tell them this will replace your role in the diagnostic procedure because they used to diagnose the cases with their eyes (visual inspection) so we would like to say this is going to support your decision in the diagnostic procedure as it quantifies your visual inspection but not totally replace your decision about the case.

This approach can be very useful in malignant diseases where the degree of overlap between the textural parameters is small not as in the diffuse liver diseases. This approach also can be useful to use it for kidney, spleen, breast, and thyroid.

This method of classification also can be used to subclassify different degrees of liver cirrhosis and quantify the turbidity of liver ascites so as to use it in clinical operations and procedures.

ACKNOWLEDGMENT

The authors are much indebted to Mr. Salim M. Emara for taking part and assistance in this work.

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